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Milan S. Matijević · Milić R. Stojić · Kurt Schlacher

# Absorption principle in process control applications

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Abstract The broad class of industrial processes has similar dynamical behavior that may be described by simple mathematical models with the dead time. The most popular, a very effective and usual structure with a long dead time compensator in use today, is the Smith predictor. However, during the last 20 years, three principal problems of the Smith predictor controlling structures have been analyzed by many authors: (1) the robustness, (2) the disturbance rejection possibilities, and (3) the extension of the idea of the Smith predictor to the case of integrative plants. Furthermore, in order to effectively use control in industrial applications, simple tuning procedures must be developed. The mentioned problems may be solved more successfully than before by use of internal model principle and control together (IMPACT) structure. In this paper, the previous modification of the Smith predictor based on the IMPACT structure is improved and generalized for process control applications with the long dead time. The crucial part of the structure synthesis is implementation of the absorption principle that is derived and implemented in the general case of the continuous SISO systems with the dead time. The structure enables the extraction of the known class immeasurable disturbances and easy setting of controller parameters in order to achieve robust stability and

M. S. Matijević (🖂)

Faculty of Mechanical Engineering, Department for Applied Mechanics and Automatic Control, University of Kragujevac, Sestre Janjić 6, 34000 Kragujevac, Serbia and Montenegro E-mail: mmatijevic@ptt.yu Tel.: +381-34-335990 Fax: +381-34-333192

M. R. Stojić Faculty of Electrical Engineering, University of Belgrade, Kralja Aleksandra 73, 11000 Belgrade, Serbia and Montenegro E-mail: estojic@etf.bg.ac.yu

K. Schlacher

Institute of Automatic Control and Control Systems Technology, Johannes Kepler University, Altenbergerstrasse 69, 4020 Linz, Austria E-mail: kurt.schlacher@jku.at performance. Both analytical analysis and simulation results show that tuning of the proposed structures is extremely simple due to relatively small number of tuning parameters, all having clear physical meanings.

**Keywords** IMPACT structure · Internal Model Principle (IMP) · Time-delay systems · Robust process control

#### Abbreviations

IMP	Internal model principle
IMC	Internal model control
IMPACT	Internal model principle and control together

### **1** Introduction

Many physical systems, such as thermal processes, chemical processes, systems having transportation or diffusion, long transmission lines in pneumatic systems etc., contain time delays. The delays cause systems to destabilise or degrade their feedback performance [1]. The risk of instability or performance degradation is expressed more if the time-delay is comparable to, or greater than, the dominant process time constants [2]. Conventional controllers, like the PID controllers, could be used when the dead-time is small, but they show poor performance when the process exhibits long deadtimes because a significant quentity of detuning is required to maintain closed-loop stability. Therefore, several methods have been suggested to deal with such processes. The Smith predictor is a simple solution to this problem and is used to improve the performance of the classical PID controller for plants with time delay [3]. Attention has been paid to this control structure over the years, but many researchers pointed out that the Smith predictor is very sensitive to modeling errors. The most sensitive parameters are the time delay and the steady-state gain of the process [1]. The modified Smith predictor has been proposed by several authors: (a) some of them focused their attention on the study of autotuning and adaptive structures, and (b) others focused their attention on the study of robust control structures [1–8]. Although the Smith predictor has the capability of transforming a time-delay control design to a delay-free problem, three principal problems of the Smith predictor structure were analyzed by many authors during the last 20 years [3-7]: (1) the robustness; (2) the disturbance rejection characteristics, and (3) the extension of the idea of the Smith predictor to the case of integrative plants. An effective answer to these issues has just been given by the structure proposed in [4], which is based on the Internal model principle and control together (IMPACT) structure [9]. The proposed structure can be interpreted as a new structure of the modified Smith predictor for processes that can be described by an integrator, a velocity gain, and a long effective transport lag. The structure enables absorption of the arbitrary class of deterministic disturbances and can be easily tuned to achieve the desired speed of set-point response and to maintain the preferred system robustness with respect to interval changes and/or uncertainties of plant parameters. This paper summarizes, expands, and generalizes the previous results, and proposes more general robust continuous controller design for processes with dead times.

Namely, there are two main aims of this paper: (1) to expand result exhibited in [4] in the case of non-integrative plants and to obtain a general controlling structure applicable to a plant with long dead times; (2) to obtain a more complete method of interpretation and application of absorption principle within continuous systems with the dead time, which generalizes and improves previous efforts exhibited in [4,9,10] and their references. The proposed controlling structure is based on the IMPACT structure, or using the internal model principle (IMP) and the internal model control (IMC) together. The most important problem in continuous control systems with long dead time consists in the IMP implementation. The IMP and absorption principle are based upon the same fundamental idea of an inclusion disturbance model into the system controlling structure. In the case of the IMPACT structure, the model of external disturbance is implicitly incorporated into the minor local loop of the control system in order to suppress or to eliminate completely the influence of immeasurable disturbance on the steady-state value of system controlled variable [9,10]. In this paper, the absorption principle and its implementation within the SISO continuous systems with the dead time will be considered more general than before, using two most typical plant models from the process industry. Contrary to the previous consideration, the implementation of absorption principle and interaction of its absorption effects, within different controlling loop, will be commented, too. Generally, the proposed structure will exclude the effects of a known class of external immeasurable disturbance on the controlled variable and will improve the system robustness. The proposed structures will enable the set point transient response and speed of disturbance rejection to be adjusted independently by setting a small number of parameters having clear physical meanings. A systematic recipe of the controller tuning will be given. The efficiency and robust properties of the proposed structure will be verified and tested by simulation.

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#### 2 Plant models and control system structure

In most cases, it is possible to find two kinds of typical processes in the industry: the ones that can be modeled by a static gain  $K_p$ , a dead-time L, and a time constant T.

$$W^{o}(s) = \frac{K_{p} e^{-Ls}}{Ts+1} = G_{p}(s) e^{-Ls}$$
(1)

and the ones that can be described by an integrator, a velocity gain  $K_v$ , and a dead-time L.

$$W^{o}(s) = \frac{K_{v} e^{-Ls}}{s} = G_{p}(s) e^{-Ls}$$
 (2)

In both cases,  $G_p(s)$  represents the delay-free part of the process, and both nominal models ((1) and (2)) can be considered as simplification of more accurate models

$$W(s) = \frac{K_p}{(T_1s+1)(T_2s+1)\cdots(T_ns+1)} e^{-s\tau}$$

$$W(s) = \frac{K_v}{s(T_1s+1)(T_2s+1)\cdots(T_ns+1)} e^{-s\tau}$$
(3)

Practically, the model is just a simplification of the real system and the artificial model elements do not necessarily have a one-to-one correspondence in the real system. Furthermore, the identified parameters may possibly vary depending on the operating point, and the model describes the dynamic behavior of the real system only to a certain degree. The controller must thus be robust and be able to deal with these constraints (it must be robust enough to allow for considerable parameter variation and model uncertainty).

The IMPACT control structure of the modified Smith predictor is shown in Fig. 1. The control portion within the system structure in Fig. 1 comprises the Smith predictor internal controller, in the main loop, and two internal models, in the local minor loop: the internal nominal plant model explicitly and the internal model of external disturbance d(t) embedded implicitly into predictive filter A(s)/C(s). Both the internal nominal plant model and disturbance model are treated as the disturbance estimator. Really, disturbance estimator estimates the influence of generalized disturbance  $\phi 1$  that comprises the influence of the external disturbance d and the influence of uncertainties of plant parameters on the system output. Uncertainties of plant modeling may be adequately described by the additive bound of uncertainties  $\overline{l}_a(\omega)$ 

$$W(j\omega) = W^{o}(j\omega) + l_{a}(j\omega), \quad |l_{a}(j\omega)| \le l_{a}(\omega).$$
(4)

The controlling structure has two control loops that can be designed independently. The minor loop compensates the influence of the generalized disturbance and increases robust system performance. The minor local control loop is designed by the proper choice of polynomials A(s) and C(s). Polynomial A includes implicit disturbance model, while the choice of C affects the speed of disturbance rejection, system



Fig. 1 IMPACT structure of the modified Smith predictor with oneinput internal nominal plant model

robustness, and sensitivity with respect to measuring noise. Good filtering properties and the system efficiency in disturbance rejection are mutually conflicting requirements. The lower bandwidth of the A(s)/C(s) filter corresponds to a higher degree of system robustness and vice versa. The dynamic of the minor local control loop within the low frequency band is described by a nominal plant model. Thus, the main control loop "sees" the minor control loop as the nominal plant model and determines the dynamic behavior of the closed loop system. In the main control loop, main controller  $G_r(s)$  will be determined to achieve the desired system set point response.

For the integrative plant (2), the proportional main controller

$$G_r(s) = K_r \tag{5}$$

may be applied. In that case, under nominal conditions, the closed-loop transfer function Y(s)/R(s) and Y(s)/D(s) will be

$$\frac{Y(s)}{R(s)} = \frac{K_r K_v}{s + K_r K_v} e^{-Ls}$$
(6)

and

$$\frac{Y(s)}{D(s)} = \frac{\left(1 - \frac{1}{R(s)} \frac{A(s)}{C(s)} \frac{K_v}{s} e^{-Ls}\right) K_v \left(1 + K_r \frac{K_v}{s} \left(1 - e^{-Ls}\right)\right)}{s + K_r K_v} \times e^{-Ls}$$
(7)

In the virtue of (6), the speed of set-point response can be adjusted by choosing appropriate values of controller gain  $K_r$  or dominant time constant  $T_r = 1/(K_r K_v)$ . According to [4], the proper choice of R(s) is  $R(s) = K_v$ . Since term  $(1 - e^{-Ls})/s$  in the numerator of the closed-loop system transfer function (7) has the frequency characteristics of zero-order hold, the speed of disturbance transient response is governed by the roots of characteristic equation

$$(s + K_r K_v)C(s) = 0 \tag{8}$$

If polynomial *C* is chosen as  $C(s) = (T_0s + 1)^n$ , lower order *n* and smaller value of  $T_o$  will correspond to a faster rejection of disturbance and a lower degree of system robustness and vice versa [4]. For the sake of simplicity and easier physical realization, it is usually assumed that n = 2. Then, we can

distinguish two tuning parameters:  $T_r$  and  $T_o$  for simple and straightforward adjustment of the set point transient response, speed of absorption of disturbance influences on steady state process output value, and degree of system robustness. This is accomplished independently; first by choice of an appropriate value of  $T_r$  and then by setting of tuning parameter  $T_o$ .

In the non-integrative plant (1) case, in order to achieve the desired closed-loop transfer function Y(s)/R(s)

$$G_{\rm de}(s) = \frac{Y(s)}{R(s)} = \frac{1}{T_r s + 1} e^{-Ls}$$
(9)

may be chosen the PI main controller (Fig. 1)

$$G_r(s) = K\left(1 + \frac{1}{T_i s}\right) = \frac{Ts + 1}{K_p T_r s}, \quad \left(K = \frac{T}{K_p T_r}, T_i = T\right)$$
(10)

and in that case

$$\frac{Y(s)}{D(s)} = \frac{\left(1 - \frac{1}{R(s)} \frac{A(s)}{C(s)} \frac{K_p}{T_{s+1}} e^{-Ls}\right) K_p T_r s \left(1 + \frac{1 - e^{-Ls}}{T_r s}\right)}{(T_r s + 1)(T s + 1)} e^{-Ls}$$
(11)

Note that the system with the non-integrative process can be commented in the similar manner as the system with the integrative process. In the case of the non-integrative plant, the proper choice of R(s) is  $R(s) = K_p$ . The gain and integral time of the PI controller have clear physical meanings [see (10)], and the parameter  $T_r$  defines the set-point response [see (9)]. The speed of disturbance transient response is defined by the roots of characteristic equation

$$(T_r s + 1)(T s + 1)C(s) = 0$$
(12)

Practically, besides identified plant parameters, there will be two tuning parameters ( $T_r$  and parameter of polynomial C) for setting the set-point response, the speed of disturbance rejection, and the system robustness. The disturbance absorption and robustness will be commented in detail in the next sections.

#### **3** Absorption principle

The absorption principle is basically identical to IMP, and its intention is including the disturbance model in the control algorithm in order to suppress or reject disturbance influence on steady-state value of the process output [9].

Namely, the broad class of continuous functions can be presented as a solution of homogeneous differential equations. It is a basic result for analog absorption filter synthesis and absorption principle implementation in the control systems. For the class of continuous signals defining the expected type of disturbances, an absorption filter  $\Phi(s)$  may be designed, whose steady-state response on specified class of signal will be equal to zero. For example, the absorption filter  $\Phi(s) = s$ 

is suitable for the class of step disturbances,  $\Phi(s) = s^2$ is suitable for ramp disturbances,  $\Phi(s) = s^2 + \omega^2$  corresponds to  $d(t) = Asin\omega t$ , etc. Generally, let us suppose that the class of disturbances d(t) has the Laplace transform  $D(s) = d_{num}(s)/d_{den}(s)$ . Then, the absorption polynomial  $\Phi(s)$  can be determined explicitly by

$$\Phi(s) = d_{\text{den}}(s), \quad D(s) = \frac{d_{\text{num}}(s)}{d_{\text{den}}(s)}$$
(13)

The principle of absorption means a design of absorption filter whose input is disturbance signal in order to compensate disturbance influence. By implementation of an absorption filter in the control system, the disturbance model is included in the controlling structure, too. The following compensation equation may be considered as the absorption condition of the given class of disturbances

$$\frac{Y(s)}{D(s)} = \frac{\Phi(s)N_d(s)}{D_d(s)} \tag{14}$$

where the polynomials  $N_d(s)$  and  $D_d(s)$  form a stable transfer function  $N_d(s)/D_d(s)$  having less or more influence on the quality of the disturbance transient response. Let us now consider the applications of the absorption principle in the process control with the dead time.

The principle of absorption in the IMPACT structure is implemented in the minor loop that enables estimation of influence of generalized disturbance, its prediction, and feedforward compensation. Using the absorption principle in the case of integrative plant and IMPACT controlling structure (see [7] and [14]), the absorption condition becomes

$$\frac{(sR(s)C(s) - K_vA(s)e^{-Ls})}{sR(s)C(s)(s + K_rK_v)}K_v\left(1 + K_rK_v\frac{1 - e^{-Ls}}{s}\right)e^{-Ls} = \frac{\Phi(s)}{D_d(s)}N_d(s)$$
(15)

Since the term  $(1 - e^{-Ls})/s$  has the known characteristics of the zero-order hold, then the transfer function

$$\left(1+K_rK_v\frac{1-\mathrm{e}^{-Ls}}{s}\right)$$

is stable and it can be consider as factor of the polynomial  $N_d(s)$ . In order for the transfer function  $N_d(s)/D_d(s)$  to be stable, it is necessary to adopt the following form of the polynomial A(s)

$$A(s) = sA_o(s) \tag{16}$$

As it is known,  $R(s) = K_v$ , and the relation (15) is reduced to

$$\frac{(C(s) - A_o(s)e^{-Ls})}{C(s)(s + K_r K_v)} K_v \left(1 + K_r K_v \frac{1 - e^{-Ls}}{s}\right) e^{-Ls} = \frac{\Phi(s)}{D_d(s)} N_d(s)$$
(17)

From (17), it is obvious that the speed of disturbance absorption is defined by the roots of characteristic Eq. (8), and that the absorption condition becomes

$$A_o(s)e^{-Ls} + N_1(s)\Phi(s) = C(s)$$
(18)

The solutions of (18) are the polynomials  $A_o(s)$  and  $N_1(s)$ , while the stable polynomial C(s) is chosen freely previously. The selection of polynomial C(s) can be done according to the desired speed of the disturbance rejection, filter system properties, and degree of the system robustness.

But, in contrary to the discrete case where Diophantine equation is solvable without any approximation, the Eq. (18) has to be reduced into polynomial equation. The exponential term  $e^{-Ls}$  can be approximated by the Pade approximation, or by the Taylor series expansion as

$$e^{-Ls} \cong 1 - Ls + \frac{(Ls)^2}{2!} - \frac{(Ls)^3}{3!} + \dots + \frac{(Ls)^N}{N!} = \sum_{k=0}^N \frac{(-Ls)^k}{k!}$$
(19)

Substituting  $e^{-Ls}$  from (19) into (18), relation (18) obtains the specific form of the Diophantine equation

$$A_o(s) \sum_{k=0}^{N} \frac{(-Ls)^k}{k!} + N_1(s)\Phi(s) = C(s)$$
(20)

A single solution of the Diophantine equation, which plays a crucial role in the design procedure of the proposed disturbance estimator, does not exist [11]. The relation (20) is a linear equation in the polynomials  $A_o(s)$  and  $N_1(s)$ . Generally, the existence of the solution of the Diophantine equation is given in [11,12]. According to [11,12], there always exists the solution of (20) for  $A_o(s)$  and  $N_1(s)$  if the greatest common factor of polynomials  $\sum_{k=0}^{N} (-Ls)^k / k!$  and  $\Phi(s)$ divides polynomial C(s); then, the equation has many solutions. The particular solution is constrained by the fact that control law must be causal, i.e.

$$\deg(A(s)) = 1 + \deg(A_0(s)) \le \deg(C(s))$$

Hence, after choosing a stable polynomial C(s), N, and degrees of polynomials  $A_o(s)$  and  $N_1(s)$ , and inserting the absorption polynomial  $\Phi(s)$  that corresponds to an expected external disturbance, polynomials  $A_o(s)$  and  $N_1(s)$  are calculated by equating coefficients of equal order from the leftand right-hand of Eq. (20). In our case, for the absorptional polynomial

$$\Phi(s) = s^m, \quad (m = 1, 2, 3...)$$
(21)

that corresponds to the class of polynomial disturbances

$$d(t) = \sum_{i=1}^{m} d_i t^{i-1}$$

and for chosen polynomial  $C(s) = c_o + c_1 s + c_2 s^2 + c_3 s^3 + \dots$ , the simplest solution of the Diophantine Eq. (20) is given in Table 1. Practically, most frequent disturbances may be

Table 1 Implicit disturbance model in general choice of polynomial C(s)

Class of disturbance	Polynomial $A_o(s)$
Step, $m = 1$	$A_o(s) = c_o$
Rampa, $m = 2$ Parabolic, $m = 3$	$A_o(s) = c_o + (c_1 + Lc_o)s$ $A_o(s) = c_o + (c_1 + Lc_o)s$
,	$+(c_2+c_1L+0.5c_o)s^2$

considered as slow varying, and in these cases, the polynomial A(s) should be calculated to correspond to the ramp signal  $d(t)(\Phi(s) = s^2, m = 2)$ . Hence, in the majority of practical applications, the appropriate choice of the absorption filter might be  $\Phi(s) = s^2, m = 2$ , which corresponds to the absorption of linear (ramp) disturbance, but, it also enables the extraction of disturbances that vary slowly, and even suppression of the effects of the low frequency stohastic disturbances.

For the sake of clarity and to reduce the number of adjustable parameters, let us assume

$$C(s) = (T_0 s + 1)^n \tag{22}$$

Then, from Table. 1 it can be calculated  $A_o(s) = 1 + (nT_0 + L)s$ , and the transfer function inside the disturbance estimator becomes

$$\frac{1}{R(s)}\frac{A(s)}{C(s)} = \frac{1}{K_v}\frac{s(s(nT_o + L) + 1)}{(T_o s + 1)^n}$$
(23)

Value of parameter *n* is constrained by the condition of causality n > 2.

In the case of non-integrative plant (1), the similar approach to the absorption principle implementation may be applied. For the sake of simplicity and correctness of relation (12), let us assume

$$A(s) = (Ts + 1)A_o(s)$$
(24)

Then, analogous to the relation (15), the compensation equation becomes

$$\frac{s\left(C(s) - A_o(s)e^{-Ls}\right)}{C(s)(T_r s + 1)(T s + 1)} K_p T_r \left(1 + \frac{1 - e^{-Ls}}{T_r s}\right) e^{-Ls} = \frac{\Phi(s)}{D_d(s)} N_d(s)$$
(25)

From (25), it is obvious that the speed of disturbance rejection is defined by the roots of characteristic Eq. (12), and the absorption condition becomes

$$s\left(C(s) - A_o(s)e^{-Ls}\right) = N_1(s)\Phi(s)$$
(26)

But, by selection of the PI controller (10) within the main control loop, the absorption of the step disturbance is already designed through the main control loop ( $\Phi_{ml}(s) = s$ ). The absorption principle in the IMPACT structure is implemented in the inner loop, but generally the disturbance absorption can be achieved by main ( $\Phi_{ml}(s)$ ) and inner ( $\Phi_{il}(s)$ ) control loop together. In our case

$$\Phi(s) = \Phi_{\rm ml}(s)\Phi_{\rm il}(s), \quad \Phi_{\rm ml}(s) = s \tag{27}$$

where  $\Phi_{ml}(s)$  and  $\Phi_{il}(s)$  are absorption polynomials defining absorption by the main and inner control loop, respectively. From here on, the relation (26) is reduced to

$$A_o(s)e^{-Ls} + N_1(s)\Phi_{il}(s) = C(s)$$
(28)

By using the Taylor series expansion of  $e^{-Ls}$  and by substituting from (19) into (28), the relation (28) becomes the Diophantine equation

$$A_0(s)\sum_{k=0}^N \frac{(-Ls)^k}{k!} + N_1(s)\Phi_{\rm il}(s) = C(s)$$
<sup>(29)</sup>

which is the same form as (20), and which guaranty the absorption of a disturbances class specified by the absorption filter  $\Phi_{il}(s)$ . The previous comments about choosing a stable polynomial C(s) and Table 1. are also applicable for solving the Diophantine Eq. (29). For example, by choosing  $C(s) = (T_0s + 1)^n$  and  $\Phi_{il}(s) = s$  (i.e.  $\Phi(s) = s^2$ ), the transfer function inside the disturbance estimator becomes

$$\frac{1}{R(s)}\frac{A(s)}{C(s)} = \frac{1}{K_p}\frac{Ts+1}{(T_os+1)^n}$$
(30)

or in the case of the parabolic disturbances ( $\Phi(s) = s^3$ ,  $\Phi_{il}(s) = s^2$ )

$$\frac{1}{R(s)}\frac{A(s)}{C(s)} = \frac{1}{K_p}\frac{(Ts+1)((nT_o+L)s+1)}{(T_os+1)^n}$$
(31)

The value of the parameter *n* is constrained by the condition of causality  $(R(s) = K_p, n \ge \deg(A(s)))$ .

# 4 Robustness analysis

The design of the controller is based on the nominal model  $W^o(s)$ , but the true open-loop transfer function is W(s). The closeness of the nominal plant  $W^o(s)$  and real plant W(s) may be described by the relation (4) and by the additive bound of uncertainty  $\bar{l}_a(\omega)$ . The real plant is considered as a member of the infinite family of plants within which each member more or less deviates from the nominal plant. Let us suppose that  $W^o(s)$  and W(s) have the same number of unstable poles and that the desired closed-loop system transfer function  $G_{de}(s)$  is stable. Then, each member of the family is stable if and only if the following criterion of robust stability is satisfied

$$\bar{l}_a(\omega) < \beta(\omega) \tag{32}$$

where

$$\beta(\omega) = \left| \frac{W^o(j\omega)}{G_{\rm de}(j\omega)} \right| \left| \frac{G_{\rm ff}(j\omega)}{G_{\rm fb}(j\omega)} \right|$$
(33)

while  $G_{\rm ff}(s)$  and  $G_{\rm fb}(s)$  are defined from

$$U(s) = G_{ff}(s)R(s) - G_{fb}(s)Y(s)$$
(34)

as the transfer functions of feedforward and feedback portions of the system control structure, respectively.

In the case of integrative plant and the IMPACT controlling structure of Fig. 1, one obtains

$$\beta(\omega) = K_v \left| \frac{T_r j \omega + 1}{j \omega} \right|$$

$$\times \left| \frac{C(j \omega)}{C(j \omega) + (T_r j \omega + 1 - e^{-Lj \omega}) A_o(j \omega)} \right| \quad (35)$$

The linear models of the finite orders fairly well, approximate dynamic behavior of plants at low frequency range, while disagreements appear at high frequencies. It is significant to notice that  $\beta(\omega)$  leads to a constant value at high frequencies. Namely, if one chooses polynomial  $C(s) = (T_0s + 1)^n$  and  $A_o(s) = a_{m-1}s^{m-1} + \cdots + a_1s + a_o(A(s) = sA_o(s))$ , then

$$\lim_{\omega \to \infty} \beta(\omega) = \frac{K_v T_r T_o^n}{T_o^n + T_r a_{m-1}}, \quad \text{for deg } C(s) = \deg A(s)$$
(36a)

$$\lim_{\omega \to \infty} \beta(\omega) = K_v T_r, \quad \text{for deg } C(s) > \deg A(s)$$
(36b)

From (36) and the previous one, one can conclude that the suitable choice of parameter n may be adopted

$$n = 1 + \deg A(s) \tag{37}$$

It is evident that a greater value of  $T_r = 1/(K_r K_v)$  yields a higher degree of the system robustness. The influence of the disturbance observer on the system robustness will be illustrated by the illustrative example in the section that follows. Generally, it will be shown that for a higher degree *n* of chosen polynomial C(s) and a greater value of time constant  $T_o$ of C(s) (see [22]), the system robustness improves and vice versa. Furthermore, the implementation of more complicated disturbance models within polynomial A(s) means a higher degree of A(s) and less system robustness.

In the case of non-integrative plant and the IMPACT controlling structure of Fig. 1, defined with the relations (1), (9), (10), (24) and  $R(s) = K_p$ , one derives

$$\beta(\omega) = K_p \left| \frac{T_r j \omega + 1}{T j \omega + 1} \right| \\ \times \left| \frac{C(j\omega)}{C(j\omega) + (T_r j \omega + 1 - e^{-Lj\omega}) A_o(j\omega)} \right|$$
(38)

It is evident that for the defined structures of both plant cases with the dead time, the influence of the minor local control loop on the system robustness is the same. Similarly to the previous one, respecting the choice of  $A_o(s) = a_{m-1}s^{m-1} + \cdots + a_1s + a_o$ , (22), (24), and (37), one derives

$$\lim_{\omega \to \infty} \beta(\omega) = \frac{K_p T_r}{T}, \quad \text{for deg } C(s) > \deg A(s)$$
(39)

It is clear that to improve the system robustness, the speed of set point response must be slowed down (i.e. desired time constant  $T_r$  must be increased). When controlling processes with long dead times, a general rule used in the process industry is that the closed-loop time constant  $T_r$  is chosen near the open-loop time constant T [4]. The relations (38) and (39) confirm this rule.

#### 5 Controller tuning and simulation results

The control part of the IMPACT structure of the modified Smith predictor in Fig. 1 contains five parameters  $K_v$ , L,  $K_r$ ,  $T_o$ , and n in the case of integrative plant, and six parameters  $K_p$ , T, L,  $T_r$ ,  $T_o$ , and n in the case of non-integrative plant. Plant parameters  $K_v$  and L, or  $K_p$ , T, and L, are measured or estimated by a simple experiment. Other parameters  $K_r$ ,  $T_o$ , and n, or  $T_r$ ,  $T_o$ , and n are to be adjusted with respect to prescribed speeds of a set-point transient and disturbance transient responses and to desired degree of system robustness. Practically, the parameter n may be fixed by (37), and then both of the structures have the same tuning parameters  $T_r$ and  $T_o$  (in the first case  $T_r = 1/(K_r K_v)$ ) with clear physical meaning. By increasing time constants  $T_r$  and  $T_o$  the system robustness and the system filter properties will be improved and, at the same time, the disturbance rejection and set point response will be slower. The time constant  $T_o$  does not influence the set point response. First, by tuning  $T_r$ , the set-point response and robust stability area may be set; and then, by tuning of  $T_o$ , the system robust performance and speed of disturbance rejection may be influenced. The efficiency of the proposed structures and procedures of the parameter tuning will be investigated by a simulation.

Let us consider particular example of the processes given by [3]

$$W(s) = \frac{0.1e^{-8s}}{s(1+s)(1+0.5s)(1+0.1s)}$$
(40)

with identified nominal plant model

$$W^{o}(s) = \frac{0.1e^{-9.7s}}{s} \tag{41}$$

and

$$W(s) = \frac{e^{-10s}}{(1+s)(1+0.6s)(1+0.15s)(1+0.1s)}$$
(42)

with the identified nominal plant model

$$W^o(s) = \frac{e^{-10.5s}}{1.5s+1} \tag{43}$$

In both plant cases, within the IMPACT controlling structure, the disturbance observer is applied with an implicit model of ramp disturbances (the relations (23) and (31)). The main controller parameters:  $T_r = 2$ ,  $K_v = 0.1$ , L = 9.7in the integrative plant case, and  $T_r = 1.5$ ,  $K_p = 1$ , T =1.5, L = 10.5 in the non-integrative plant case, are chosen. The influence of the disturbance observer (23) and (31) on robust stability is illustrated in Fig. 2. In the virtue of Fig. 2, for a higher degree *n* of the chosen polynomial C(s)and a greater value of time constant  $T_o$ , the system robustness improves. The efficiency of the IMPACT structure is illustrated in Figs. 3 and 4. In all simulation runs, the reference is  $r(t) = 0.5 \cdot 1(t)$ , and disturbance is the same, marked by d(t). Figure. 3 explains the capability of the IM-PACT structure (Fig. 1) in the integrative plant case. First,



Fig. 2 Influence of disturbance observer parameters on the robust stability -1 T<sub>o</sub> = 9, 2 T<sub>o</sub> = 6, 3 T<sub>o</sub> = 3



Fig. 3 The disturbance absorption in the case of IMPACT structure with integrative plant and an impicit model of step (y1) and ramp (y2) disturbance

the structure is designed to absorb a constant disturbance  $(n = 2 \text{ and } T_o = 1)$  and trace y1(t) is obtained. Second, the structure is designed to absorb a ramp disturbance by using transfer function (23), with n = 2 and  $T_o = 6$  (and trace y2(t) is obtained). Generally, the design of the local minor loop for the absorption of a more complex external disturbances d(t) requires a higher order of polynomial  $A_o(s)$ , and results in a lower degree of robustness. Because of that, the similar level of robust stability is reached by different values  $T_o$ , as it is shown in example in Fig. 4.



Fig. 4 The response of the structure with non-integrative plant and PI main controller (10) with (y1) and without (y2) local minor loop

Figure 4 explains the capability of the IMPACT structure (Fig. 1) in the non-integrative plant case. Trace y2(t) of Fig. 4 shows the reference and the disturbance response of the structure in Fig. 1 with the main controller (10), but without the local minor loop. Then, the proposed IMPACT structure for a non-integrative plant (1) is designed to absorb a ramp disturbance. More precisely, the disturbance observer for a step disturbance absorption (30), with n = 2 and  $T_o = 1.5$ , is implemented in the controlling structure, and trace y1(t)is obtained. Notice that each linear segment of the disturbance is absorbed after a certain time period. The disturbance rejection may be improved by choosing n = 1 and/or a smaller values of  $T_o$  and  $T_r$ . However, in doing so, one must maintain the robust stability with respect to the uncertainties of the plant parameters.

## **6** Conclusion

The most common design goal in the process control is to obtain a critically damped closed-loop system which is as fast as possible, with a possibility to take into account the model of uncertainties and to tune their characteristics with respect to set points and disturbances. In order to meet these requirements, the usage of the absorption principle and the modified IMPACT structure with a simple and robust tuning is proposed. The analysis is made using the two typical plant models with delays that are found in the process industry. Practically, the results from [4] are expanded and generalized on both non-integrative and integrative plants, while a formulation and use of absorption principle in the SISO continuous systems are given in more detail than before. The exhibited discussion about the synthesis, implementation, and interaction of the absorption filters within the control system, explains and improves the absorption principle use for the general case of the continuous SISO system synthesis (with or without dead times, with different structure, with integrative or non-integrative plant, etc.). For both cases (of integrative and non-integrative plant), the particular controller forms that enable mutual analog effects and common tuning rules are given. The tuning of the modified IMPACT structures is discussed in the paper and some simple rules are proposed. The proposed structures may be adjusted according to the desired speed of set-point response and speed of disturbance rejection, in a simple way by tuning only few parameters having clear physical meanings. In both cases, the structure can be easily tuned manually. The robustness and response speed are mutually opposite requirements. However, the proposed structure is suitable for successful design of the robust stability and robust performance, and for rejection of influence of arbitrary external disturbance class at same time. Generally, the structure enables further improvements: on-line system adaptation, combination of advantages of approved control algorithms, etc. Several simulation results are presented to verify previous theory analysis and to illustrate the structure efficiency.

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